

Chapter 8 — Systems of Equations

Teacher Guide

Teacher Chapter-at-a-Glance

Materials

Graph paper and a ruler. A calculator is optional for the decimal-rate taxi problem in Worksheet 3.

Pacing

Spend the least time on graphing (students already have this from Chapter 6) and the most time on classification (no solution vs. infinitely many).

Talk Prompt

"Did the variables actually cancel, and if so, is what's left true or false?"

If Ready-to-Learn Reveals a Gap

If students cannot solve a two-step equation or substitute a value into an equation, reteach the relevant part of Chapter 3 or 4 before continuing.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Giving Only One Coordinate (Mistake 1). *States a single value with no matching second coordinate as the final answer to a system.* Review before the next chapter.

Missed Tripwire #2 → Confusing No Solution With Infinitely Many Solutions (Mistake 6). *After the variables cancel, cannot say whether the resulting statement is true or false, or what each case means.* Review before the next chapter.

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. Solve $x + 7 = 15$.

Correct answer: $x = 8$

Likely wrong answer: $x = 22$

Misconception: Adds 7 to both sides instead of subtracting it.

Next move: Review inverse operations: undo addition with subtraction.

2. Is (3, 4) on the line $y = x + 1$?

Correct answer: Yes, since $4 = 3 + 1$.

Likely wrong answer: No.

Misconception: Substitutes only one coordinate, or checks it against the wrong side of the equation.

Next move: Review substituting both coordinates of a point into an equation to test it.

3. Solve $2x + 3 = 11$.

Correct answer: $x = 4$

Likely wrong answer: $x = 7$

Misconception: Subtracts 3 but forgets to then divide by 2, treating the last step as addition instead.

Next move: Review two-step equations: undo addition or subtraction first, then undo multiplication or division.

Common Mistakes

TEACHING NOTE

Mistakes 5 and 6 (parallel lines, and telling a contradiction from an identity) are the highest-value reteaching targets. Most Tripwire #2 misses trace back to one of these two.

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Mistake 1: Giving Only One Coordinate

What students do: A student solves a system and writes $x = 4$ as the final answer.

Why this happens: Students are used to solving one-variable equations, where a single number is the whole answer.

What to do instead: A system with x and y needs an ordered pair. Substitute the x -value back in to find y , then write both coordinates.

Diagnostic trigger: States a single value with no matching second coordinate as the final answer to a system.

Mistake 2: Checking Only One Equation

What students do: A student checks the answer in only one of the two original equations.

Why this happens: Students think one successful check is enough proof.

What to do instead: The solution must satisfy both equations. Check both originals, every time.

Diagnostic trigger: Shows a check against only one equation, with no verification against the second.

Mistake 3: Substitution Without Brackets

What students do: Substituting $y = x - 3$ into $2x - 4y = 10$ is written as $2x - 4x - 3 = 10$.

Why this happens: The student forgets that the whole expression $(x - 3)$ replaces y , not just the x .

What to do instead: Write $2x - 4(x - 3) = 10$ first, with brackets around the whole substituted expression, then distribute.

Diagnostic trigger: Substitutes a multi-term expression for a variable without enclosing it in brackets first.

Mistake 4: Losing a Sign During Elimination

What students do: $(2x + 3y) - (2x - y)$ is simplified to $2x$.

Why this happens: Students forget to distribute the negative sign to every term of the second equation.

What to do instead: Distribute first: $2x + 3y - 2x + y = 4y$. The sign on every term of the subtracted equation flips.

Diagnostic trigger: A subtraction between two equations only flips the sign of the first term, not every term.

Mistake 5: Treating Parallel Lines as One Solution

What students do: A student sees two equations with the same slope and tries to force an intersection point.

Why this happens: Students assume every system must have exactly one ordered-pair solution.

What to do instead: Same slope with different y-intercepts means the lines are parallel. There is no solution.

Diagnostic trigger: Keeps solving after recognizing two lines share a slope, instead of stopping to classify the system.

Mistake 6: Confusing No Solution With Infinitely Many Solutions

What students do: A student thinks $14 = 15$ and $14 = 14$ mean the same thing.

Why this happens: Students see the variables disappear either way and are not sure what the leftover statement means.

What to do instead: A false statement (like $14 = 15$) means no solution. A true statement (like $14 = 14$) means infinitely many solutions.

Diagnostic trigger: After the variables cancel, cannot say whether the resulting statement is true or false, or what each case means.

Practice

Teacher solutions follow this practice set.

1. What does the solution of a system of two linear equations represent on a graph?
2. Solve $y = x + 3$ and $y = -x + 9$.
3. Solve $y = 3x - 2$ and $x + y = 10$.
4. Solve $x + y = 11$ and $x - y = 1$.
5. Solve $2x + y = 13$ and $x - y = 2$.
6. Solve $3x + 2y = 18$ and $x + y = 7$.
7. Classify $y = -2x + 8$ and $y = -2x - 1$.
8. Classify $x + 3y = 12$ and $2x + 6y = 24$.
9. Solve $y = 4x + 1$ and $y = -x + 11$.
10. Solve $2x + 5y = 20$ and $3x - 5y = 5$.

11. Two numbers have a sum of 26 and a difference of 8. Find the numbers.

12. Compare $C = 4k + 6$ and $C = 2k + 18$. When do the costs match?

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. What does the solution of a system of two linear equations represent on a graph?

Solution: The solution is the intersection point: an ordered pair that satisfies both equations.

2. Solve $y = x + 3$ and $y = -x + 9$.

Solution: Set $x + 3 = -x + 9$: $2x = 6$, so $x = 3$ and $y = 6$. Solution: (3, 6).

3. Solve $y = 3x - 2$ and $x + y = 10$.

Solution: Substitute $y = 3x - 2$ into $x + y = 10$: $4x - 2 = 10$, so $x = 3$ and $y = 7$.

4. Solve $x + y = 11$ and $x - y = 1$.

Solution: Add $x + y = 11$ and $x - y = 1$: $2x = 12$, so $x = 6$ and $y = 5$.

5. Solve $2x + y = 13$ and $x - y = 2$.

Solution: Add $2x + y = 13$ and $x - y = 2$: $3x = 15$, so $x = 5$ and $y = 3$.

6. Solve $3x + 2y = 18$ and $x + y = 7$.

Solution: Use $y = 7 - x$ in $3x + 2y = 18$: $x + 14 = 18$, so $x = 4$ and $y = 3$.

7. Classify $y = -2x + 8$ and $y = -2x - 1$.

Solution: The lines have the same slope but different intercepts, so they are parallel and have no solution.

8. Classify $x + 3y = 12$ and $2x + 6y = 24$.

Solution: The second equation is twice the first, so both describe the same line and there are infinitely many solutions.

9. Solve $y = 4x + 1$ and $y = -x + 11$.

Solution: Set $4x + 1 = -x + 11$: $5x = 10$, so $x = 2$ and $y = 9$.

10. Solve $2x + 5y = 20$ and $3x - 5y = 5$.

Solution: Add the equations: $5x = 25$, so $x = 5$. Then $2(5) + 5y = 20$ gives $y = 2$.

11. Two numbers have a sum of 26 and a difference of 8. Find the numbers.

Solution: Let the numbers be x and y : $x + y = 26$ and $x - y = 8$. Then $2x = 34$, so the numbers are 17 and 9.

12. Compare $C = 4k + 6$ and $C = 2k + 18$. When do the costs match?

Solution: Set $4k + 6 = 2k + 18$: $2k = 12$, so $k = 6$ and both costs are \$30.

Tripwire Check

These questions test whether the thinking is solid — not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. A student solves a system and writes $x = 4$. What is incomplete about this answer?

Correct answer: The y-coordinate is missing. A solution to a two-variable system must be an ordered pair.

Likely wrong answer: $x = 4$ is already complete.

Misconception: The student is treating a two-variable system like a one-variable equation.

Next move: Substitute $x = 4$ into one of the original equations to find the matching y-value.

2. A student eliminates a variable and reaches $18 = 21$. What does this mean?

Correct answer: No solution. The statement is false, so the lines are parallel and never intersect.

Likely wrong answer: Keep solving to find a value for x.

Misconception: The student assumes every system must have an ordered-pair solution, and does not recognize a false numeric statement as a valid, meaningful result.

Next move: Connect the false statement directly to parallel lines. Contrast with a true identity, which means infinitely many solutions.

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. **Two lines intersect at (2, 5). What does this point represent for the system?**

1. It is the solution: the point that makes both equations true. Likely wrong answer: says it's just where the lines cross, with no mention of making both equations true.

Misconception: The intersection point is described visually without connecting it to satisfying both equations.

2. **Two lines have the same slope but different y-intercepts. How many solutions does the system have?**

2. No solution. Likely wrong answer: infinitely many solutions. Misconception: Same slope is treated as meaning the lines are identical rather than merely parallel.

3. **Two equations graph as the exact same line. How many solutions does the system have?**

3. Infinitely many solutions. Likely wrong answer: no solution. Misconception: Two equations describing the same line are mistaken for two parallel lines that never meet.

4. Solve $y = -2x + 5$ and $x - 2y = -5$.

4. (1, 3) Likely wrong answer: substitutes y into the first equation instead of the second. Misconception: The substitution is placed back into the equation it came from instead of the other one.

5. Solve $y = x + 6$ and $2x + y = 12$.

5. (2, 8) Likely wrong answer: (8,2) (swaps which value is x and which is y). Misconception: The two coordinates of the solution are reported in the wrong order.

6. Solve $y = 3x - 1$ and $y = -x + 11$.

6. (3, 8) Likely wrong answer: sets the equations equal but solves for y first instead of x . Misconception: Solving proceeds in a different order than the setup requires, leading to a mismatched pair.

7. Solve $x + y = 13$ and $x - y = 1$.

7. (7, 6) Likely wrong answer: subtracts the equations instead of adding, losing the y -terms incorrectly. Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

8. Solve $3x + y = 16$ and $3x - y = 8$.

8. (4, 4) Likely wrong answer: subtracts the equations instead of adding, failing to eliminate y . Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

9. Solve $2x + y = 16$ and $x - y = 2$.

9. (6, 4) Likely wrong answer: subtracts the equations instead of adding, failing to eliminate y . Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

10. Is (2, 3) a solution to $x + y = 5$ and $2x - y = 1$? Show both checks.

10. Yes. Both checks work. Likely wrong answer: checks only one of the two equations. Misconception: A solution is confirmed against a single equation instead of both.

11. A movie ticket costs twice as much as a snack. Together they cost \$15. Let m be the ticket price and s be the snack price. Write a system for this situation.

11. $m + s = 15$; $m = 2s$. Likely wrong answer: $m = 2s$ written as $s = 2m$ (reverses which price is double the other). Misconception: The relationship between the two prices is reversed when writing the equation.

12. Solve the system from Question 11.

12. $m = \$10$; $s = \$5$. Likely wrong answer: $m = \$5$, $s = \$10$ (swaps the two prices). Misconception: The two solved values are assigned to the wrong variable.

13. Why is $x = 4$ not a complete answer for a system with x and y ?

13. The y -coordinate is missing. Likely wrong answer: says it's fine since x is the 'main' variable. Misconception: A solution is assumed complete once one coordinate is found, without recognizing a system needs both.

14. Classify $3x + y = 10$ and $6x + 2y = 20$.

14. Infinitely many solutions. Likely wrong answer: no solution (doesn't check whether the equations are true multiples of each other). Misconception: A repeated equation in disguise is mistaken for two contradictory ones.

15. Classify $x + 2y = 6$ and $2x + 4y = 15$.

15. No solution. Likely wrong answer: infinitely many solutions (doesn't check whether the constants actually match after scaling). Misconception: Two equations with proportional variable terms but different constants are mistaken for identical lines.

Worksheet B: Independent Practice

1. Solve $y = 2x + 4$ and $3x + y = 19$.

1. (3, 10) Likely wrong answer: (10, 3) (swaps x and y in the solution). Misconception: The two coordinates of the solution are reported in the wrong order.

2. Solve $x + y = 15$ and $x - y = 1$.

2. (8, 7) Likely wrong answer: adds the equations but mishandles the sign, getting the wrong sum for x . Misconception: Elimination is attempted but a sign is dropped when the equations are combined.

3. Solve $4x + y = 23$ and $x - y = 2$.

3. (5, 3) Likely wrong answer: subtracts the equations instead of adding, failing to eliminate y . Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

4. Adult tickets cost \$15 and student tickets cost \$9. A total of 12 tickets are sold for \$156. Find each number.

4. 8 adult tickets; 4 student tickets. Likely wrong answer: sets up $15a + 9s = 12$ and $a + s = 156$ (swaps which number is the price and which is the count). Misconception: The price-per-ticket and the number-of-tickets roles are assigned to the wrong equation.

5. Solve $y = -x + 12$ and $x + 2y = 15$.

5. (9, 3) Likely wrong answer: (12, 0) (misreads the substitution and drops a term). Misconception: A term is dropped when substituting one equation into the other.

6. Solve $3x + 2y = 20$ and $x + y = 8$.

6. (4, 4) Likely wrong answer: subtracts the equations instead of using elimination correctly, mismatching coefficients. Misconception: Elimination is attempted without first matching a coefficient in both equations.

7. Classify $y = 5x + 1$ and $y = 5x - 9$.

7. No solution. Likely wrong answer: infinitely many solutions (misreads equal slopes as identical lines). Misconception: Equal slopes with different intercepts are mistaken for the same line instead of parallel ones.

8. Solve $2x + y = 17$ and $2x - y = 7$.

8. (6, 5) Likely wrong answer: subtracts the equations instead of adding, failing to eliminate y . Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

9. Solve $2x + 3y = 19$ and $x - y = 2$.

9. (5, 3) Likely wrong answer: adds the equations directly without matching a coefficient first. Misconception: Elimination is attempted without first scaling one equation to match a coefficient.

10. A store sells binders for \$4 and folders for \$1. A customer buys 18 items for \$42. Find each number.

10. 8 binders; 10 folders. Likely wrong answer: sets up $4b+1f=18$ and $b+f=42$ (swaps which number is the price and which is the count). Misconception: The price-per-item and the number-of-items roles are assigned to the wrong equation.

11. Solve $y = 4x - 5$ and $2x + y = 13$.

11. (3, 7) Likely wrong answer: substitutes y into the same equation it came from. Misconception: The substitution is placed back into the equation it came from instead of the other one.

12. Solve $5x + y = 26$ and $2x - y = 9$.

12. (5, 1) Likely wrong answer: subtracts the equations instead of adding, failing to eliminate y . Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

13. Classify $3x - y = 6$ and $6x - 2y = 12$.

13. Infinitely many solutions. Likely wrong answer: no solution (doesn't check whether the second equation is a multiple of the first). Misconception: A repeated equation in disguise is mistaken for two contradictory ones.

14. Solve $2x + y = 11$ and $3x - 2y = 6$.

14. (4, 3) Likely wrong answer: multiplies the first equation by the wrong factor before eliminating. Misconception: The scaling factor chosen to match coefficients for elimination doesn't actually make them equal.

15. Two numbers have a sum of 40 and a difference of 14. Find the numbers.

15. 27 and 13. Likely wrong answer: writes $40-14=26$ and treats that as one of the numbers directly. Misconception: The sum and difference are combined incorrectly instead of set up as two separate equations first.

16. Solve $y = 2x + 3$ and $y = -3x + 18$.

16. (3, 9) Likely wrong answer: sets the equations equal but solves for y first, producing a mismatched pair. Misconception: Solving proceeds in a different order than the setup requires, leading to a mismatched pair.

17. Solve $x + 3y = 17$ and $2x - y = 6$.

17. (5, 4) Likely wrong answer: subtracts the equations instead of matching a coefficient first. Misconception: Elimination is attempted without first scaling one equation to match a coefficient.

18. Classify $2x + 4y = 10$ and $x + 2y = 7$.

18. No solution. Likely wrong answer: infinitely many solutions (doesn't check whether the constants actually match after scaling). Misconception: Two equations with proportional variable terms but different constants are mistaken for identical lines.

19. Plan A costs \$18 plus \$7 per class. Plan B costs \$42 plus \$3 per class. When do the plans cost the same?

19. 6 classes. Likely wrong answer: sets $18 + 7c = 42 + 3c$ up backwards, swapping which plan has the higher flat fee. Misconception: The flat fee and per-class rate for the two plans are assigned to the wrong plan.

20. Choose substitution or elimination for $x = 2y + 1$ and $3x + y = 17$. Explain briefly, then solve.

20. Substitution, since x is already isolated; (5, 2). Likely wrong answer: chooses elimination and introduces an unnecessary extra step. Misconception: The more efficient method (substitution, since a variable is already isolated) is not recognized as the better choice.

Worksheet C: Mixed Application

1. A theatre sells adult tickets for \$14 and child tickets for \$9. It sells 16 tickets for \$189. How many of each type were sold?

1. 9 adults; 7 children. Likely wrong answer: sets up $14a + 9c = 16$ and $a + c = 189$ (swaps which number is the price and which is the count). Misconception: The price-per-ticket and the number-of-tickets roles are assigned to the wrong equation.

2. A farm has chickens and goats. There are 23 animals and 70 legs. How many chickens and goats are there?

2. 11 chickens; 12 goats. Likely wrong answer: sets up $\text{legs} + \text{animals} = 70$ and $\text{chickens} + \text{goats} = 23$ without distinguishing 2 legs vs 4 legs per animal. Misconception: The different number of legs per animal type is not built into the equation.

3. A taxi costs $C = 2.25k + 5$. Another costs $C = 1.50k + 11$. After how many kilometres do the costs match?

3. 8 km. Likely wrong answer: sets the two cost expressions equal but solves for C instead of k. Misconception: The variable representing distance is not isolated; a different variable is solved for instead.

4. Solve $y = 3x + 2$ and $2x + y = 22$. Then check both equations.

4. (4, 14) Likely wrong answer: checks only one of the two equations. Misconception: A solution is confirmed against a single equation instead of both.

5. Solve $4x - y = 12$ and $x + y = 8$. Then state the method used.

5. (4, 4); elimination. Likely wrong answer: subtracts the equations instead of adding, failing to eliminate y. Misconception: Elimination is attempted with the wrong operation for the sign pattern present.

6. Classify $2x + y = 9$ and $4x + 2y = 18$. Explain what the graph looks like.

6. Infinitely many solutions; same line. Likely wrong answer: no solution (doesn't check whether the second equation is a multiple of the first). Misconception: A repeated equation in disguise is mistaken for two contradictory ones.

7. Classify $x - y = 4$ and $2x - 2y = 11$. Explain what the graph looks like.

7. No solution; parallel lines. Likely wrong answer: infinitely many solutions (doesn't check whether the constants actually match after scaling). Misconception: Two equations with proportional variable terms but different constants are mistaken for identical lines.

8. Two numbers have a sum of 54. One number is three times the other. Find both.

8. 13.5 and 40.5. Likely wrong answer: sets up $x+y=54$ and $3x=y$, then solves for the wrong variable as 'the other number'. Misconception: The two unknowns are mixed up when reporting which value is three times the other.

9. A museum charges \$11 for adults and \$6 for students. Thirty visitors pay \$235. How many adults and students attended?

9. 11 adults; 19 students. Likely wrong answer: sets up $11a+6s=30$ and $a+s=235$ (swaps which number is the price and which is the count). Misconception: The price-per-person and the number-of-people roles are assigned to the wrong equation.

10. Solve $y = -2x + 15$ and $3x + y = 17$.

10. (2, 11) Likely wrong answer: substitutes y into the same equation it came from. Misconception: The substitution is placed back into the equation it came from instead of the other one.

11. Solve $2x + 5y = 31$ and $3x - 5y = 14$.

11. (9, 13/5) Likely wrong answer: adds the equations directly without checking that the y -terms are already opposite. Misconception: Elimination is attempted with an unnecessary extra scaling step, introducing an arithmetic slip.

12. A student says the solution to $y = x + 4$ and $y = x - 2$ is (0, 4). Is the student correct? Explain.

12. No. Parallel lines never intersect, so there is no solution at all. Likely wrong answer: (0,4) is the solution. Misconception: Two lines with the same slope are assumed to intersect somewhere, when parallel lines with different intercepts never meet.

13. Write a system with solution (4, -1). Check it in both equations.

13. Answers vary. Example: $y = x - 5$ and $x + y = 3$, checked at (4, -1). Likely wrong answer: writes two equations that don't actually share the point (4,-1). Misconception: An equation is written to look right without actually checking that the target point satisfies it.

14. Non-routine diagnostic: a student substitutes $y = x - 3$ into $2x - 4y = 10$ and writes $2x - 4x - 3 = 10$. Identify the error, correct the substitution, and solve.

14. Correct substitution is $2x - 4(x - 3) = 10$; solution (1, -2). Likely wrong answer: (already given) $2x - 4x - 3 = 10$. Misconception: The substituted expression is inserted without brackets, so only part of it receives the intended multiplication.

15. Non-routine diagnostic: one system simplifies to $0 = 0$. Another simplifies to $0 = 7$. Explain what each result means on a graph.

15. $0 = 0$ means infinitely many solutions (same line). $0 = 7$ means no solution (parallel lines). Likely wrong answer: (already given, explanatory item). Misconception: A variables-cancel result is treated as an error to keep re-solving instead of a meaningful outcome to interpret as true or false.

Answer Key — Exit Ticket

1. (4, 9)
2. (4, 2)
3. No solution

Worksheet 1: Guided Practice

1. It is the solution: the point that makes both equations true.
2. No solution.
3. Infinitely many solutions.
4. (1, 3)
5. (2, 8)
6. (3, 8)
7. (7, 6)
8. (4, 4)
9. (6, 4)
10. Yes. Both checks work.
11. $m + s = 15$; $m = 2s$.
12. $m = \$10$; $s = \$5$.
13. The y-coordinate is missing.
14. Infinitely many solutions.
15. No solution.

Worksheet 2: Independent Practice

1. (3, 10)
2. (8, 7)
3. (5, 3)
4. 8 adult tickets; 4 student tickets.
5. (9, 3)
6. (4, 4)
7. No solution.
8. (6, 5)
9. (5, 3)
10. 8 binders; 10 folders.

11. (3, 7)
12. (5, 1)
13. Infinitely many solutions.
14. (4, 3)
15. 27 and 13.
16. (3, 9)
17. (5, 4)
18. No solution.
19. 6 classes.
20. Substitution, since x is already isolated; (5, 2).

Worksheet 3: Mixed Practice & Application

1. 9 adults; 7 children.
2. 11 chickens; 12 goats.
3. 8 km.
4. (4, 14)
5. (4, 4); elimination.
6. Infinitely many solutions; same line.
7. No solution; parallel lines.
8. 13.5 and 40.5.
9. 11 adults; 19 students.
10. (2, 11)
11. (9, 13/5)
12. No. Parallel lines never intersect, so there is no solution at all.
13. Answers vary. Example: $y = x - 5$ and $x + y = 3$, checked at (4, -1).
14. Correct substitution is $2x - 4(x - 3) = 10$; solution (1, -2).
15. $0 = 0$ means infinitely many solutions (same line). $0 = 7$ means no solution (parallel lines).